Revealing the nature of the final image in Newton's *experimentum crucis*

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In his crucial prism experiment, Newton noted the position of the final image, but not its shape or coloring. Most scholars describe the image as a single-colored representation of the selective aperture; some report multiple colors. When the experiment is re-enacted as the transformation of a *camera obscura* image, it becomes clear that the final image is a rainbow-colored representation of the outside world. Backward ray tracing enhances Newton's demonstration of diverse refrangibility. Using a projector, teachers can easily bring this historical experiment into the classroom and build a bridge to modern applications in hyperspectral imaging and spectral encoding.
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I. INTRODUCTION

Newton's *experimentum crucis*[1–5] is a well-known optics experiment;[6–18] see Fig. 1, cf. Table I and Fig. 2. Newton's first step was to refract a narrow beam of sunlight with a prism, obtaining a long, rainbow-colored image.[1,3,5] Newton understood this spectrum as a vertical series of interlocking solar images.[19] He concluded that sunrays are diverse in the degree to which they are refracted.[1,5] To prove this *diverse refrangibility*, Newton took a second step: Turning the first prism to and fro, he passed different parts of the spectrum through a selective aperture and a second prism. Looking for diverse refrangibility, Newton observed the final image only with regard to its *diverse positions*;[1,5] its shape and coloring were irrelevant to him. In a letter to Pardies,[2] Newton wrote that the final image was more or less in the shape of the selective aperture, but he left the exact shape "to be determined by geometricians."[3] Both in his "New Theory"[1] and in his *Opticks*,[4] the shape and coloring of the final image are not described.

Scholars[11,20–24] and textbook authors[25–27] who describe the *experimentum crucis* usually believe that the final image is a single-colored representation of the selective aperture. They claim that the colors of the spectrum are not split up by the second prism. This scientific belief[14] has been handed down from Newton's generation to ours.[12,18] However, some scientists have observed multiple colors behind the second prism.[28–30] When Newton was challenged to explain these colors, he replied that diverse rays come from diverse parts of the sun, and that additional rays could come from clouds.[31] Arguing about the colors of rays, Newton and his critics ignored the shape of the final image.

All in all, descriptions of the final image have been incomplete and inconsistent. Our purpose here is to clarify the shape and coloring of the final image. To this end, we will reconsider the entire imaging process.

Newton's ray drawings[2,5] indicate this imaging process (Fig. 1): Sunrays are refracted, selected, and refracted again. Still, the shape of the final image is unclear because the third dimension is missing. We cannot even assign colors to the rays because Newton did not draw to scale. To achieve the correct proportions, we would need a sketch that is almost life-sized. Thus, a ray-based approach seems impractical.

Instead, we will use an image-based approach,[9,32,33] which is outlined in Sec. II. In Secs. III and IV, we apply this approach to the *experimentum crucis*. In Sec. V, we turn to an equivalent experiment to validate our view of the final image. In Sec. VI, the teacher will find ways to bring the *experimentum crucis* into the classroom, and in Sec. VII, we highlight the results and implications of our image-based approach.

II. PRINCIPLES OF AN IMAGE-BASED APPROACH

To enjoy the benefits of an image-based approach, we will follow its three major principles (cf. Refs. 9, 32, and 33):

Principle I: *Watch* how the phenomenon changes as the experimental conditions change.
Principle II: *Think* in terms of images instead of light rays.
Principle III: *Draw* rays only to represent lines of sight, not trajectories of light.

If we apply the first principle successfully, our insights will be systematic and based on visual experience. Yet how can we change the experimental conditions without changing Newton's experimental setup? We will achieve this by starting with only one optical element, and then adding the other elements, one by one. Newton's description starts with prism *P1* (Fig. 2), but we will start differently.[34]

For a direct application of the second principle, we will begin with the pinhole H to produce a concrete image. Then, we will put the other elements at their designated places, watching how they successively change the initial image into the final image. Thus, we will see more than Newton saw, and gain a holistic understanding of the imaging process. Even Newton thought in terms of images when explaining the spectrum,[19] but he thought in terms of rays when discussing the light behind the selective aperture.[1,3,5,31]

The third principle allows us to interpret the imaging process in terms of rays, without speculating about the nature of light.[9] However, before we introduce any rays, we will think everything through in terms of images.

III. FROM INITIAL TO FINAL IMAGE

To understand the shape and coloring of the final image, we will re-enact Newton's *experimentum crucis* as the creation and transformation of a *camera obscura* projection.

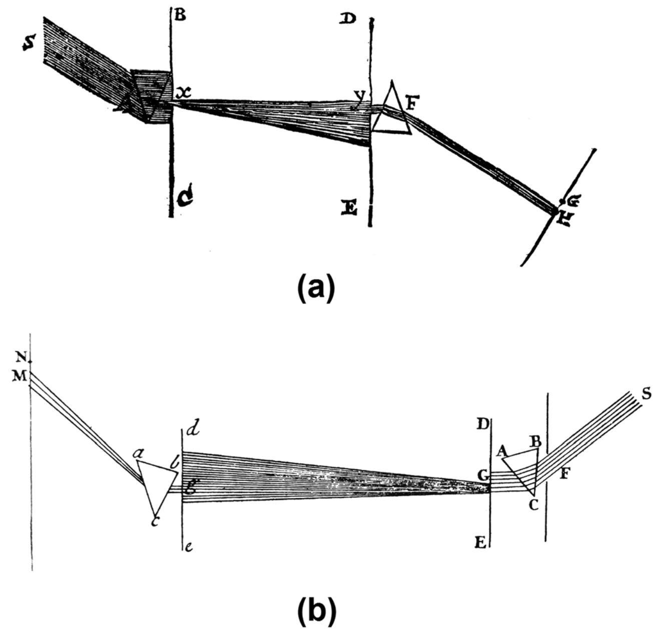

Fig. 1. Newton's drawings of his *experimentum crucis*. (There is no such drawing in his "New Theory" (Ref. 1).) For the notation, see Table I. (a) From a letter to Pardies, 1672 (Ref. 2). (Read drawing from left to right.) The prisms are mutually inverted; the final rays hardly diverge. (b) From *Opticks*, 1704 (Ref. 5). (Read drawing from right to left.) The prisms are non-inverted; the final rays clearly diverge.

Table I. Notation for optical elements, cf. Figs. 1 and 2.

Optical element	Newton 1672 (Ref. 2)	Newton 1704 (Ref. 5)	Grusche
First prism	*A*	*ABC*	*P1*
Pinhole	*x* (in *BC*)	*G* (in *DE*)	*H*
Selective aperture	*y* (in *DE*)	*g* (in *de*)	*A*
Second prism	*F*	*abc*	*P2*

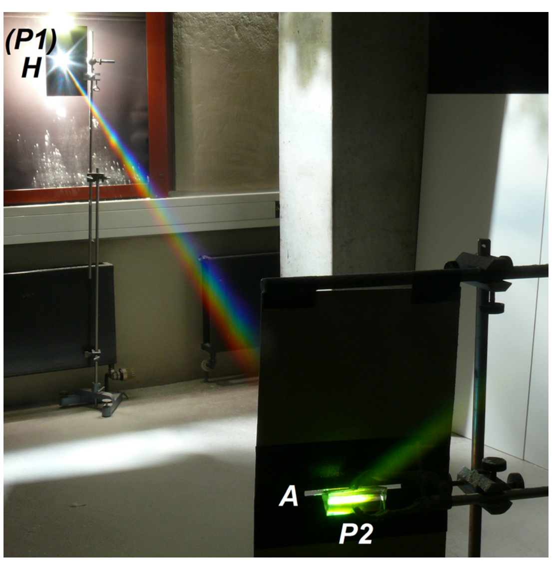

Fig. 2. (Color online) Unenhanced photo of the setup for Newton's *experimentum crucis*. The first prism *P1*, hidden by the board with pinhole *H*, is in the position of minimum deviation, as in Fig. 1(b), so that the rainbow-colored spectrum is projected to the lowest possible position. The selective aperture *A* and the second prism *P2* are 12 feet away from *P1* and *H* (Refs. 1 and 5). (To reveal the refracted light beams, flour was thrown into the air.)

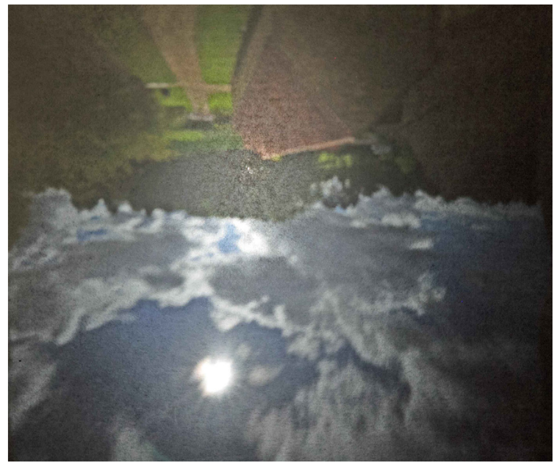

Fig. 3. (Color online) Newton's room-sized *camera obscura* produces an upside-down image of the Woolsthorpe countryside. Original projection on translucent paper, photographed from behind with a Canon EOS 7D; exposure time: 8 s at *f*/14.

A. Through the pinhole: *Camera obscura* projection

Having darkened his study room, Newton makes a wide hole in the window shutter and puts behind it a board pierced with a pinhole.[1,3,5] Literally, he has turned his room into a *camera obscura* (Latin for dark chamber), or pinhole camera. Realizing that each of our eyes is a *camera obscura*, too,[17] we can understand what is projected inside Newton's chamber: an upside-down representation of the view from the pinhole (Fig. 3). Although we can see trees, meadows, houses, and the sky, Newton mentions only the solar disk[1] [Fig. 4(a)].

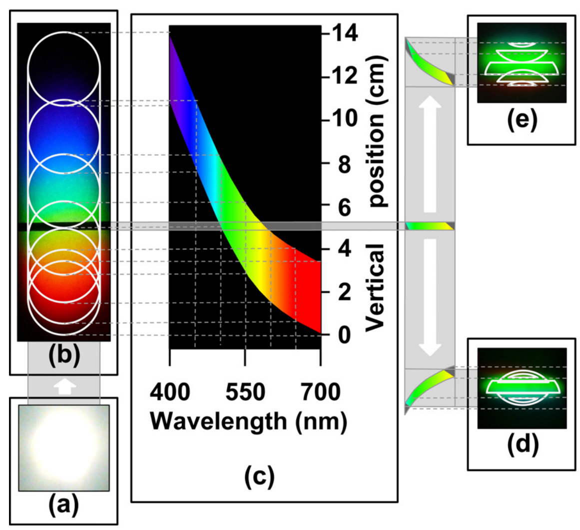

Fig. 4. (Color online) Image transformation in Newton's *experimentum crucis*. (a) The pinhole makes an image of the solar disk [cf. Fig. 3]. The first prism spreads the white circle into a series of differently colored circles (Ref. 19). Different circles contribute different horizontal strips of the solar disk to the selective aperture, here a horizontal slit. (c) Plotting circle position versus wavelength, we obtain a dispersion diagram (for the symmetry axis of the spectrum) (Ref. 35). The selected segment of the dispersion diagram is vertically sheared upon dispersion by the second prism (Ref. 41), which is inverted in (d) and not inverted in (e). Correspondingly, the selected strips of the solar disk are shifted apart according to wavelength, forming a multicolored, elliptical image of the sun. (Newton's hole—a special case of our slit—yields a vertical strip of that image.)

B. Through the first prism: Dispersed image

Next, Newton puts a prism in front of the pinhole.[1,3,5] Still, we have a *camera obscura* projection. Again, it corresponds to the view from the pinhole.[17] From the pinhole, Newton would see the landscape through the prism, now. Thus, he would see a blurry, multicolored image of the landscape. If we look through the pinhole and prism with monochromatic filters, we see sharp images of the landscape.[35] These images are similar in shape but different in color and position. Hence, the blurry prismatic image comprises a series of sharp, monochromatic images.[35,36]

Accordingly, we interpret the sun's spectrum as a series of differently colored circles[19] [Fig. 4(b)]. Having fixed our equilateral crown-glass prism in the position of minimum deviation,[1,10,14] we trace the outline of the spectrum[37,38] on a piece of cardboard 12 feet away.[1,5] Within this outline, we trace individual images of the sun where we see them through monochromatic filters.[37] As the spectrum is moving according to the sun, we need to continually re-position the cardboard, until we have drawn the circles for all filters (for wavelengths 400 nm, 450 nm, 500 nm, 550 nm, 600 nm, 650 nm, and 700 nm). Refining the drawing in several iterations,[37] we obtain an accurate picture of how the prism shifts the solar image to different places for different wavelengths [Figs. 4(b) and 4(c)].

C. Through the selective aperture: Spectrally encoded image

Newton's projection screen for the spectrum contains a selective aperture in the form of a small hole.[1,3,5] To move the spectrum up and down across the hole, he turns the first prism to and fro. He needs to hurry[1,5] because the azimuthal motion of the sun causes its spectrum to run sideways across the hole within a few minutes.

Seeing that the horizontal position of the selective aperture is arbitrary, we will use a horizontal slit[6,12,16,17] instead of the hole. Later on, we will treat Newton's hole as a special case. Using a slit, we need not even turn the first prism: The altitudinal motion of the sun causes all parts to successively go through the slit within about ten minutes.

Remember that Newton's spectrum is a series of differently colored circles.[19] Suppose the slit admits the green part of the spectrum, as in Figs. 2 and 4(b). In this case, the greenish-blue circle (seen through the 500 nm filter) contributes its lower part to the slit, the orange circle (seen through the 600 nm filter) contributes its upper part, while intermediate circles contribute intermediate parts. On the wall behind the selective aperture, we see a single-colored slit image. Through monochromatic filters, we can see that this slit image is composed of different horizontal strips of differently colored circles.

To speak in modern terms, the whole disk of the sun is *spectrally encoded* in the slit image [cf. Figs. 4(b) and 4(c)]. Spectral encoding is the representation of one spatial dimension in the wavelength dimension. This process has been applied in lensless microscopy,[39] fiber-based endoscopy,[40] and surround-view video projection[41] (see Sec. V B). Moreover, spectral encoding underlies *spatiospectral scanning*[42]—a technique for *hyperspectral imaging*, whereby the chemical composition of an object is revealed.

D. Through the second prism: Spectrally decoded image

To refract the selected light, Newton places a second prism behind the selective aperture.[1,3,5] Directly behind the second prism, where the horizontal strips of the solar disk are still overlapping, we still see a single-colored slit image. The further we move the final projection screen from the second prism, the further the horizontal strips move apart. The strips are arranged according to wavelength. If the second prism is inverted relative to the first, as in Fig. 1(a), the strips are arranged in the original order, yielding an image of the sun that is upside-down like the initial image [Figs. 4(d) and 5(a)]. If the second prism is non-inverted, as in Fig. 1(b), the strips are arranged in reverse order, yielding a blurrier, right-side-up image [Figs. 4(e) and 5(b)]. With a non-inverted prism, the shift among the strips is equal and opposite to the shift with an inverted prism (assuming that the prism is always in the position of minimum deviation, and that the distance from the prism to the final image is the same). Yet with an inverted prism, the final image is narrower because the upper part of the lower circle and the lower part of the upper circle from the spectrum are hidden inside the final image [Fig. 4(d)]. In both cases, the final image is a multicolored, elliptical image of the sun.

The final image is especially impressive when the sun is rising or setting behind barren trees. Within about ten minutes, we can watch the solar disk wander through all the color bands, revealing the silhouettes of branches and twigs. Evidently, the final image is a rainbow-colored representation of an entire scene, the brightest part of which is the sun (cf. Appendix). If we reduce our slit[6,12,16,17] to Newton's hole,[1,3,5] we only see a vertical strip of that scene.

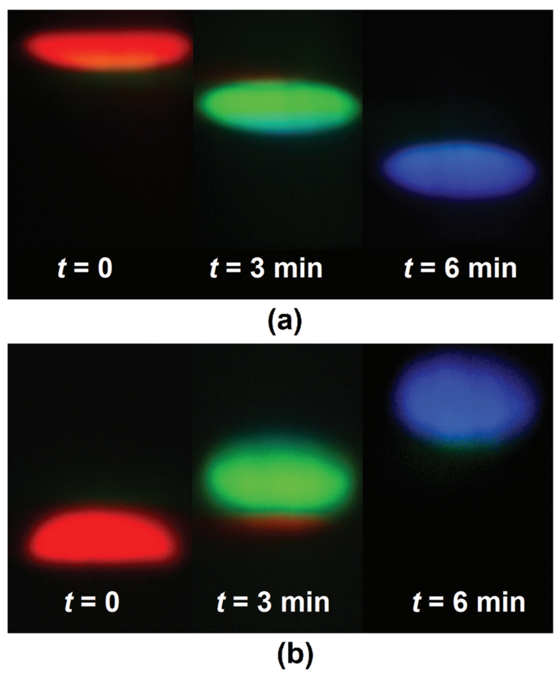

Fig. 5. Three overlaid photos of the final image at different times *t*. With a horizontal slit as the selective aperture—and with both prisms fixed—one can watch a rainbow-colored "motion picture" of the rising sun. The projection screen is vertically placed 4 feet behind the second prism. (a) With prisms inverted as in Fig. 1(a), the rising sun seems to set. (b) With non-inverted prisms, as in Fig. 1(b), the sun appears less sharp. Photographed with a Panasonic DMC-FZ50; exposure time: 2 s at *f*/11.

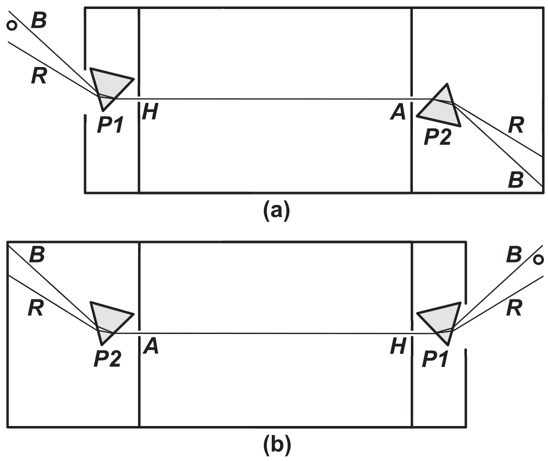

Fig. 6. Backward ray tracing (from points in the final image to the corresponding object points); (a) is the counterpart to Fig. 1(a) and (b) is the counterpart to Fig. 1(b). The blue line of sight (*B*) goes to the uppermost part of the scene, the red line of sight (*R*) goes to the lowermost part of the scene; intermediate lines (omitted) go to intermediate parts. We infer that the angle between rays *R* and *B* at *P1* equals the field of view (Ref. 42), and that rotation of *P1* effects spatiospectral scanning (Ref. 42), whereby the bright solar image is moved through the otherwise dark color bands on the final screen.

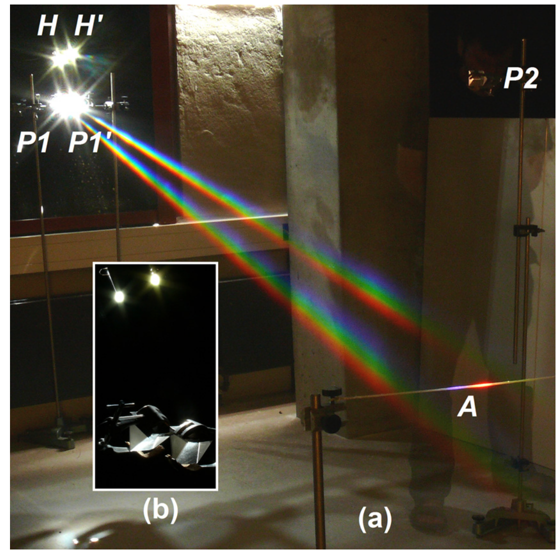

Fig. 7. (Color online) Unenhanced photo of the setup for Newton's thread experiment. (a) Pinholes *H* and *H'* each produce a *camera obscura* image of the outside world (note the double images of trees and clouds on the pillar). The two solar images are dispersed by prisms *P1* and *P1'* into two spectra. At a distance of 12 feet, the selective aperture *A*—here a thread—receives one color from each spectrum. The viewer, semitransparent in this long-exposure photo, views the thread through prism *P2*, cf. Fig. 8(b). (b) *H*, *H'* and *P1*, *P1'*.

IV. FROM IMAGES TO RAYS

Now that we have understood the *experimentum crucis* in terms of images, we may interpret it in terms of rays[9] that connect object points and image points.

Newton traced rays forward from the sun to the final image, as shown in Fig. 1. However, this mechanistic approach led him to idealize the sunrays as parallels, to ignore the sun's surroundings, and to presuppose diverse refrangibility.

For a simpler, more accurate, and less speculative approach, we exploit the *principle of reversibility* (Axiom III in *Opticks*[4]): Tracing rays backward from image points to object points, we automatically follow only the relevant rays (Fig. 6). These Euclidean lines of sight[9] directly support the Newtonian notion of differently refrangible rays. Moreover, a symmetry emerges that is missing in Newton's forward ray tracing: the final image must always be a rainbow-colored representation of the outside world.

To make the final image single-colored, Newton devised a lens-enhanced version[43] of his *experimentum crucis*.[12,18] Even in this experiment, the final image represents the outside world, as I will show elsewhere.

V. WHAT NEWTON DID NOT SEE

A. "One sees only what one knows"—Goethe

Did Newton know that the final image represents the outside world? For a definitive answer, we turn to a related experiment. In his *Opticks*, directly after the *experimentum crucis*, Newton describes a thread experiment[44] [Fig. 7, cf. Fig. 2]. He uses a white thread as the selective aperture, puts the first prism behind the pinhole and takes up the second prism to look through it toward the thread.

The thread experiment works like the *experimentum crucis* (cf. Fig. 4): The pinhole produces a white solar image; the first prism spreads the white circle out into a series of differently colored circles; the selective aperture receives different strips of these circles; the second prism shifts these image strips.[45] To compare the shift for two different colors, Newton adds a pinhole *H'* and prism *P1'* to get a second spectrum.[44]

Now that Newton's selective aperture is linear, he should see through his prism a pair of ellipses. However, he does not. He describes the final images as straight lines, and even draws them thus [Fig. 8(a)].[44] It lies in the nature of science that "meaningful observation is not possible without pre-existing expectation."[14] Apparently, Newton is blinded by his ray-based preconceptions. His supporter Desaguliers blindly follows him [Fig. 8(b)].[31] Both see the thread "divided into two."[31,44] The Royal Society agrees.[31]

With our image-based approach, we know that the final prism does more than just divide: It destroys the image of the thread, creating solar images instead (Fig. 9). These elliptical images become circular as we approach the prisms *P1* and *P1'*. Even Goethe, who repeated and criticized all of Newton's experiments, failed to see these multicolored images of the sun.[46]

B. A brighter version of Newton's thread experiment

So far, we have only seen images of the sun. Can we see sunlit objects, too? To make the final image as bright as

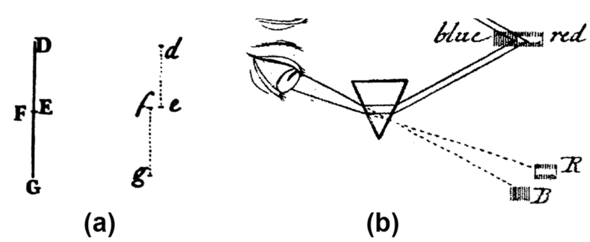

Fig. 8. Newton's and Desaguliers's misinterpretations of the final image in the thread experiment. (a) Newton sees the thread "divided into two parallel Threds" (Ref. 44). (b) Desaguliers repeats the experiment with a paper strip instead of the thread. Likewise, he sees the paper strip "divided into two" (Ref. 31).

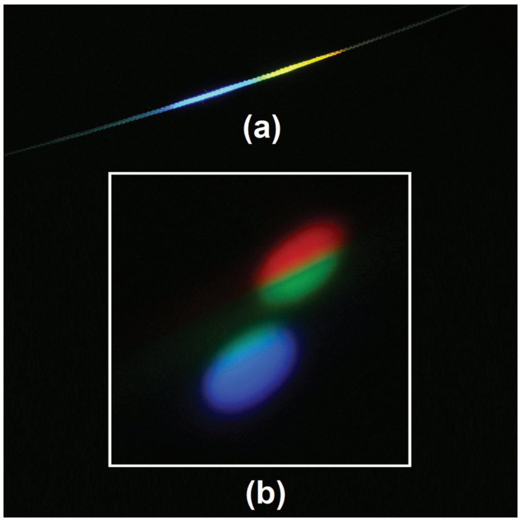

Fig. 9. Re-enactment of Newton's thread experiment (cf. Fig. 7). White thread illuminated with blue and yellow from prisms *P1* and *P1′*. (b) Viewed through prism *P2*, the final image is in the form of two ellipses, each representing the sun. (Only if the first two prisms are too far off the position of minimum deviation, the ellipses look like lines.) Photographed with a Panasonic DMC-FZ50; exposure time: 8 s at *f*/11.

possible, we bring the prisms closer to the thread, and replace the thread with a linear translucent screen to be viewed from behind (Fig. 10, cf. Ref. 41). Direct-vision prisms have a larger dispersion angle than simple ones, allowing us to double the field of view to about 5°.[41,42] As expected, the final image is a rainbow-colored image of the sunlit scene (Fig. 11).

Even Newton and Goethe—albeit with simple prisms—could have done such an experiment. However—apparently unaware that the final image represents the outside world—they did not.

VI. BRINGING THE *EXPERIMENTUM CRUCIS* INTO THE CLASSROOM

A. *Camera obscura* projection or video projection

The *experimentum crucis* is easy to re-enact in the classroom. Moreover, one can build a dollhouse version at a scale of 1:10, take it outside into the sunshine, observe the

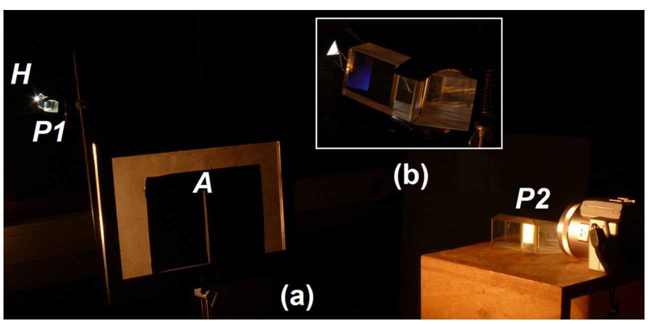

Fig. 10. (Color online) Photo of the setup for a brighter version of Newton's thread experiment [cf. Fig. 7. (a)]. The outside world is projected through pinhole *H* and direct-vision prism *P1* onto a translucent projection screen. Black paper with a 4 mm wide slit is taped onto the screen to obtain a translucent version *A* of Newton's thread. The distance from *H* to *A* is 1.5 m. Through direct-vision prism *P2*, a camera photographs toward *A* from a distance of 1 m. For maximum image sharpness, *P1* is not inverted relative to *P2* (Ref. 41). (b) Pinhole *H* (made triangular to show that the shape of the hole does not significantly affect the projected image) and prism *P1*.

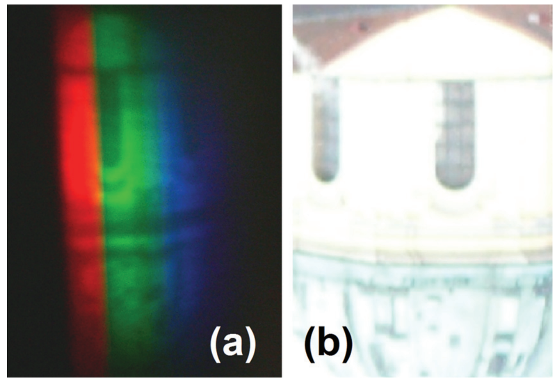

Fig. 11. Images of the basilica of Weingarten (upside-down like the initial *camera obscura* image). (a) Photo taken with a Panasonic DMC-FZ50 behind *P2* (cf. Fig. 10). Although the exposure was 60 s (at *f*/11), the spectral image itself was visible to the unaided eye, as long as the sun was shining bright. (b) Direct view (rotated).

spectrum and a pea-sized final image of the sun, and bring the dollhouse back inside. However, the *experimentum crucis* and its dollhouse version only work on sunny days.

Alternatively, teachers may simulate the *camera obscura* projection with a video projector (cf. Ref. 11). With the first prism before the projector, the rest of the setup can be a replica of the original experiment.

B. A modern application: Projected-Image Circumlineascopy (PICS)

Having replaced the scene and pinhole with a video projector, the teacher can translate the historical experiment into a modern application, called Projected Image Circumlineascopy (PICS).[41] A video is projected through the first prism and onto a white thread, or, more conveniently, an uncooked spaghetti noodle. Sitting in a circle around the spaghetti, students can view through the second prism a sharp, rainbow-colored version of the video.[41] This projection method epitomizes the nature of the final image in Newton's *experimentum crucis*: It is a rainbow-colored representation of the initial image.[47]

VII. CONCLUSION

For the first time in the discourse on Newton's *experimentum crucis*, we have revealed the nature of the final image: it is a rainbow-colored representation of the outside world. If the selective aperture is a slit, the final image is two-dimensional; if the slit is reduced to a hole, only a vertical strip of this image remains. When Newton turned his prism to and fro, he unknowingly scanned up and down the landscape before his window.

Newton interpreted the final image as a representation of the selective aperture. His friend Desaguliers, and even his enemy Goethe, perpetuated this interpretation. Evidently, Newton's conceptual change from images to rays was premature, and scientific belief hindered scientific insight. Still, our image-based approach only serves to underline, not undermine, the validity of Newton's demonstration of diverse refrangibility.

Because the final image is always rainbow-colored, the lensless setup for Newton's *experimentum crucis* is not a monochromator. Instead, the setup is an imaging system. We have identified the imaging principle as spectral coding. This

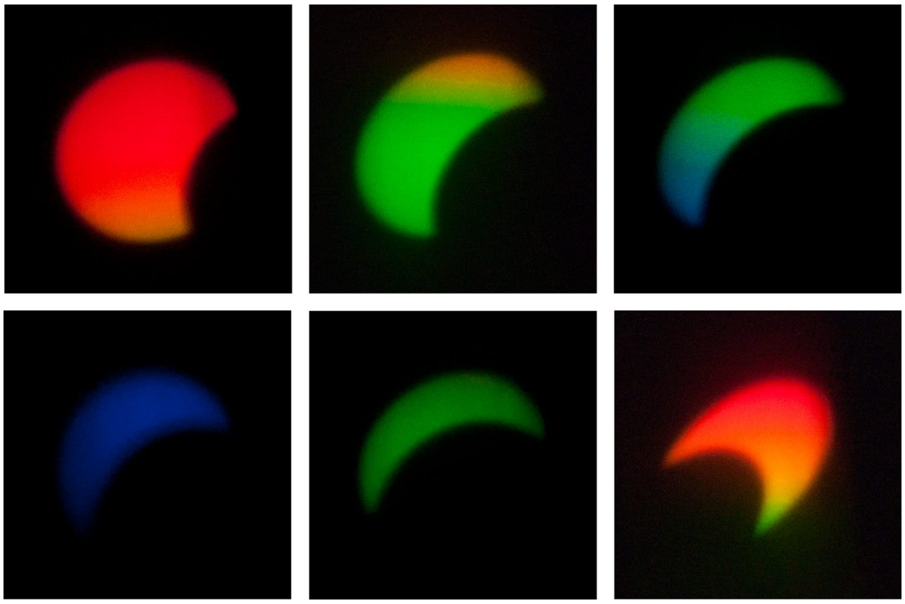

Fig. 12. (Color online) The final image in Newton's *experimentum crucis*, produced during a solar eclipse over Weingarten/Germany on 20 March 2015. (The selective aperture was a horizontal slit.) Photos taken with a Panasonic DMC-FZ50, exposure time: 1 s at *f*/3.2.

principle is nowadays used in fields such as biomedical imaging and remote sensing. Accordingly, teachers can build a bridge between the historical experiment and modern applications.

ACKNOWLEDGMENTS

In 2010, Professor Dr. Johannes Grebe-Ellis and Dr. Matthias Rang sparked my interest in the *experimentum crucis*, and Florian Theilmann invited me to explore spectra with him. In January 2014, four dozen 10th and 11th grade students from the Einstein Gymnasium in Potsdam/Germany joined my project *Newton versus Grusche*, which formed the basis of this manuscript. One of the students, Henriette Bast, helped improve the text. Ann Blackett and assistants at Woolsthorpe Manor provided the photo for Fig. 3. Alexander Neub built a dollhouse version with me. Anne Smith revised the language. The two anonymous reviewers inspired me to improve the argument and illustrations.

APPENDIX: PROJECTING A SOLAR ECLIPSE WITH THE *EXPERIMENTUM CRUCIS*

On 20 March 2015, between about 9:30 and 11:50 a.m., there was a solar eclipse over Germany. To project the partial eclipse, I used Newton's *experimentum crucis*; the images are shown in Fig. 12.

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[16]M. Rang, "Mehrfachanwendung von Spiegelspaltblenden und Prismen—eine moderne Form von Newtons *experimentum crucis*," in *Didaktik der Physik. Frühjahrstagung Bochum*, edited by H. Grötzebauch and V. Nordmeier (PhyDid, Berlin, 2009), pp. 1–12, <http://www.experimentum-lucis.de/Paper/DPG09-Tagung_Rang.pdf>.

[17]M. Rang and O. Müller, "Newton in Grönland," Philos. Nat. **46**, 61–114 (2009).

[18]Y. Takuwa, "The historical transformation of Newton's *experimentum crucis*: Pursuit of the demonstration of color immutability," Hist. Sci. **23**, 113–140 (2013); available at researchmap.jp/?action=cv_download_main&upload_id=64609.

[19]Book I, Part I, Exp. 5, Illustration in Ref. 4.

[20]According to Ref. 9, a selected ray is refracted "without any sign of dispersion."

[21]According to Ref. 14, "if one separates from the coloured spectrum a narrow beam of light, its colour will not be changed by a second prism".

[22]Grebe-Ellis identifies the hole *H* as a point source, concluding that the final projection screen should display a relatively sharp and single-colored shadow image of the selective aperture, see J. Grebe-Ellis, "Bild und Strahl in der Optik Newtons," in *Ostwalds Klassiker der exakten Wissenschaften, Band 20*, edited by J. Grebe-Ellis (Harri Deutsch, Frankfurt am Main, 2011), pp. XII–XVIII.

[23]In a video inspired by Ref. 9, the audio track pretends that "the hue remains unchanged" behind the second prism, while the film itself betrays additional hues, see Pehr Sällström, *Monochromatic Shadow Rays. A Film About Experiments on the Rehabilitation of Darkness* (edition waldorf, Stuttgart, 2010).

[24]O. L. Müller, *Mehr Licht. Goethe mit Newton im Streit um die Farben* (S. Fischer Verlag, Frankfurt am Main, 2015).

[25]Newton "showed that a small portion of the spectrum could not be spread into any other colors by passing the light through a second prism," see *Contemporary College Physics. Third Edition, 2001 Update*, edited by E. Jones and R. L. Childers (McGraw-Hill, New York, 2001).

[26]"Newton […] passed a pencil of sunlight through a prism. The emergent light fanned out into a *spectrum* or rainbow of colors …. These spectral colors were not further dispersed by additional prisms," writes M. Katz, *Introduction to Geometrical Optics* (World Scientific, Singapore, 2004).

[27]Let me translate a passage from *the* classic of German university textbooks: "The rays going [from the yellow part of Newton's spectrum through a hole and] through P2 to the second screen… were not pulled apart into a colored band again, but created only a yellow spot." The caption to the accompanying schematic reads: "Non-decomposability of color." See H.-J. Eichler, "Dispersion und Absorption des Lichtes," in *Bergmann—Schaefer Lehrbuch der Experimentalphysik, Band 3: Optik*, 10. Auflage, edited by H. Niedrig (Walter de Gruyter, Berlin, 2004), pp. 189–300.

[28]The German poet Goethe observes that the second refraction adds reddish and bluish fringes to the yellow part of the spectrum, see paragraph 137 in J. W. v. Goethe, *Farbenlehre, Band 3: Enthüllung der Theorie Newtons*, edited by G. Ott and H. O. Proskauer (Verlag Freies Geistesleben, Stuttgart, 1979).

[29]As elaborated in Refs. 12 and 18, many scholars falsely assume that the *experimentum crucis* was to prove color immutability. Newton himself stated in his lectures that he found yellow rays among the red, and blue rays among the violet. Still, Lucas was surprised to find red rays among the violet, and Mariotte, who performed a similar experiment, questioned Newton's theory when he found violet rays among the red, as well as red rays among the violet.

[30]When I involved Rang (cf. Ref. 17) in a Socratic dialogue, he characterized the final image as a blurry, multi-colored version of a part of the spectrum. Only when asked what would be *seen*, he came to the same conclusion as me, see Sec. III D.

[31]I. Newton and J. T. Desaguliers, "An account of some experiments of light and colours," Philos. Trans. **29**, 433–447 (1714).

[32]J. Grebe-Ellis, "Phänomenologische Optik: eine 'Optik der Bilder.' Teil 1: Erkenntnistheoretische, experimentiermethodische und didaktische Merkmale eines nichtreduktionistischen Zugangs zur Optik," chim. did. **32**, 137–186 (2006); available at www.physikdidaktik.uni-wuppertal.de/fileadmin/physik/didaktik/Forschung/Publikationen/Grebe-Ellis/Bartholinus_Huygens_2011.pdf.

[33]G. Maier, *An Optics of Visual Experience* (Adonis Press, Hillsdale, 2011).

[34]Arranging the optical elements in the *spatial* sequence *P1–H–A–P*2, one can choose from $N = 4! = 24$ *temporal* sequences. Ours is probably easiest to follow.

[35]F. Theilmann and S. Grusche, "An RGB approach to prismatic colours," Phys. Educ. **48**, 750–759 (2013).

[36]Newton sketches and explains this in *Optica*, Part II, Lecture 12, paragraph [104] in I. Newton, *The Optical Papers of Isaac Newton, Volume I. The Optical Lectures* 1670-1672, edited by A. Shapiro (Cambridge U.P., Cambridge, 2010).

[37]Newton traces only the *superficial structure* of the spectrum, see Book I, Part II, Exp. 7 in Ref. 4. Nonetheless, he sketches his conceptualization of the *underlying structure* as a series of circles, see note 19.

[38]With an equilateral flint-glass prism, our spectrum was 10 cm longer. On a white sheet of paper or white cloth, the spectrum was another 70% longer because the whitening agents made ultraviolet visible as blue.

[39]A. Schwarz, A. Weiss, D. Fixler, Z. Zalevsky, V. Micó, and J. García, "One-dimensional wavelength multiplexed microscope without objective lens," Opt. Commun. **282**, 2780–2786 (2009).

[40]M. Merman, A. Abramov, and D. Yelin, "Theoretical analysis of spectrally encoded endoscopy," Opt. Express **17**, 24045–24059 (2009).

[41]S. Grusche, "Spectral synthesis provides two-dimensional videos on a one-dimensional screen with $360°$-visibility and mirror-immunity," Appl. Opt. **53**, 674–684 (2014).

[42]S. Grusche, "Basic slit spectroscope reveals three-dimensional scenes through diagonal slices of hyperspectral cubes," Appl. Opt. **53**, 4594–4603 (2014).

[43]Book I, Part I, Exp. 12 in Ref. 4.

[44]Book I, Part I, Exp. 7 in Ref. 4.

[45]Inspecting the illuminated thread through a prism is analogous to projecting an illuminated slit through a prism because the retina is analogous to a projection screen and there is a non-zero distance between the prism and the projection screen, cf. Ref. 41.

[46]To explain why the blue and red parts of the thread move apart, Goethe passes several parts of the spectrum through a perforated board, seeing through his prism that the parts behave like the whole: The spectrum is stretched so that its parts are moved apart. However, in the thread experiment, even Goethe sees lines, see paragraphs 138–149 in the reference from note 28.

[47]Admittedly, the final image is also a representation of the selective aperture, but only to the extent to which a *camera obscura* projection is a representation of the pinhole.